**Module 2 Primes**

“Looking for patterns trains the mind to search out and discover the similarities that bind seemingly unrelated information together in a whole. A child who expects things to ‘make sense’ looks for the sense in things and from this sense develops understanding. A child who does not expect things to make sense sees all events as discrete, separate, and unrelated.”

Mary Bratton-Lorton

Math Their Way

**Video One**

Definition

Abundant, Deficient, and Perfect numbers, and 1, too.

Formulas for Primes – NOT!

Group?

Relatively Prime numbers

Fundamental Theorem of Arithmetic

Spacing in the number line

Lucky numbers

Popper 02 Questions 1 – 7

**Definition**

What exactly is a prime number?

One easy answer is that they are the natural numbers larger than one that are not composites. So the prime numbers are a proper subset of the natural numbers.

A prime number cannot be factored into some product of natural numbers greater than one and smaller than itself. A composite number can be factored into smaller natural numbers than itself. So here is a test for primality: can the number under study be factored into more numbers than 1 and itself? If so, then it is composite. If not, then it’s prime

A more positive definition is that a prime number has only 1 and itself as divisors. Composite numbers have divisors in addition to 1 and themselves. These additional divisors along with 1 are called proper divisors.

For example:

The divisors of 12 are {1, 2, 3, 4, 6, 12}. { 1, 2, 3, 4, 6} are the proper divisors.

A prime number’s set of proper divisors contains only 1. So this is a more positive definition but it needs some vocabulary work to make sense.

**Popper 02 Question 1**

This focus on divisors is instructive. Many mathematicians have focused on divisors and found theorems and interesting facts about divisors.

**Abundant, Deficient, and Perfect numbers, and 1, too.**

N = { {1}, perfect numbers, abundant numbers, and deficient numbers}

A **perfect** number is a natural number whose proper divisors sum back to itself.

6 is the smallest perfect number;

{1, 2, 3} are the proper divisors and these add to 6.

28 is the next perfect number;

{1, 2, 4, 7, 14} is the proper divisor set. Add them to check.

In between the perfect numbers in the number line come some deficient and abundant numbers.

A **deficient** number has proper divisors that add up to less than itself and

an **abundant** number has proper divisors that add up to more than itself.

Let’s put these on a number line:

**Popper 02 Question 2**

And sketch a diagram of this:

It is not known if there are any odd perfect numbers; we’ve only found even ones but can’t prove that perfect numbers are a proper subset of the even numbers!

Nor do we have a proof that there are an infinite number of perfect numbers on the number line. We do know that there are an infinite number of both even and odd deficient numbers. We have an estimate of how the deficient numbers are spread through the number line but we don’t know where the next perfect number is!

There are 37 known perfect numbers. The newest one was discovered on January 27, 1998. You should always be able to find the most recent information on the Internet at:

<http://www.mersenne.org/prime.htm>

Yes, there are a bunch of enthusiasts who study perfect numbers and the Mersenne Primes (next video).

Let’s look at an interesting set diagram of this partition set on top of the evens and the odds.

Let’s look at some numbers and put them in the new partition:

2, 6, 12, 13, 14, 945

Now let’s take a moment and look at the descriptors for the number 2:

Positive, negative

Even, odd

Prime, composite

Perfect, abundant, deficient

Natural, Integer, Rational, Real, Complex, Quaternion

We are building up quite a list of properties of 2.

Note that primes are a proper subset of deficient numbers!

**Popper 02 Question 2**

**Formulas for Primes – NOT!**

It would be really nice if there were a formula that would reliably produce prime numbers. For many years mathematicians tried to come up with a function that would do this. We now know that there is not a formula of any sort that will do this.

One that got a lot of attention was



It doesn’t find all the primes, but for the first 40 natural numbers it does produce primes (1, 41), (2, 43), (3, 47) and so on. But then at 41 – it gives you a composite number. Look at the formula closely:

A better formula that works until 79:

.

This one is also a parabola if you use all real numbers as the input. But

(80, 1681 = 412 ) puts an end to the dream that using natural numbers in this formula will find only primes.

Knowing these gave hope to the idea of a formula for primes, though. People are hard-wired to look for patterns and math is an expression of that characteristic!

A famous French lawyer, Pierre de Fermat was a math enthusiast and published prolifically in number theory in his spare time. We will see a many of his theorems in this class. He found that  produced primes for the first 5 whole numbers. It was later found that  = 4, 294, 967, 297 was composite.

Let’s look more closely at the formula. Note that it doesn’t have a polynomial format; it is quite different from the two above. That exponent is itself has an exponent.

We can calculate carefully: 

n = 0 (0, 3)

n = 1 (1, 5)

n = 2 (2, 17)

(3, 257) and (4, 65537)

Note that Fermat and his contemporaries in the 1600’s were doing these calculations by hand! So it’s quite understandable that they weren’t successful at factoring the fifth Fermat number.

It seems that the formula really does stop producing primes after 4. People have studied the point pairs quite a bit over the years and not found any more primes.

**Popper 02 Question 3**

**Group?**

You might think that prime numbers have the structure of a group under addition or multiplication. But they do not. Let’s look at why.

Primes: {2, 3, 5, 7, 11, 13, 17, 23…} and addition

Primes and multiplication…

If we add 3 and 5 we get 8 which is not prime. It’s not closed. If we try with multiplication, 3 times 5 is 15…which is not prime. Worse, neither 1 nor 0 is prime so the identities are not in the set. No group action with these!

What use are these things then?

**Fundamental Theorem of Arithmetic**

Well, they actually are the building blocks for the composites. It turns out that each composite number can be factored to a list of primes. And that list of primes is the only factor list that will produce that particular composite number.

First we learn 2 times 2 then 3 times that number and then 5 times that number is 60. And 60 divided by 2 is the product of the remaining list of prime factors: 30!

That is a very short summary of a learning process that takes students years to achieve. And often, students get so into the “trees” they never look at the forest. This class is really about the big picture – the forest and it’s place on the landscape.

**Relatively Prime numbers**

We compare numbers by calling them “relatively prime” if they share no common factors other than 1. This allows us to know a lot about the numbers under consideration.

For example 5 and 12 are relatively prime. The proper divisors of them are {1} and {1, 2, 3, 4, 6} respectively. When we intersect these two sets we get {1}. We symbolize this fact in typical math fashion by reusing symbols we’ve used in other contexts!

(5, 12) = 1. This reads “5 and 12 are relatively prime”.

So now we know what we have to do to add

 The common denominator will be 60, not something smaller.

If two numbers are NOT relatively prime, you can get away with a smaller number as the common denominator.

With 6 and 45, the intersection yields {3}

(6, 45) = 6

so you know you don’t need to use all the threes in the factor lists to get a least common multiple to use as a common denominator. You would use 2, 3, 3, and 5 to get the number to use rather than a number with three 3’s in the factor list.

Factor 6

Factor 45

Note the overlap!

Another example: 4 and 10. 4 factors to 2 and 2 and 10 factors to 2 and 5. If you look at the factor list you see that one of the 2’s is common to each list. We can use proper divisor sets to identify this common factor. The set of proper divisors for 4 is {1, 2} and for 10 it is {1, 2, 5}. The intersection of these sets is {2}, the common factor. (4, 10) = 2, in other words.

You have an extra 2 in the 4 and a surplus 5 in the 10 so now you can build your special “1’s” to finish the job. LCM is 20.



What is the Big Picture view of this problem?

We are illustrating that the rational numbers are closed under addition because

(Q, +) is a group.

It’s easy to get lost in the process though, if finding a common denominator is not taught in such a way that students see the factoring and the common factors as something worth remembering or tied to what they already know.

**Popper 02 Question 4**

**Spacing in the numberline**

It is instructive to look at primes in order and on a number line. Let’s start with a list of natural numbers and how they fit, in order, in the partition

{{1}, {primes}, {composites}}

1 in its own set

2 prime

3 prime

4 2x2…using 2 which is smaller

5 prime

6 2x3…using smaller primes

7 prime

8 2x2x2

9 3x3

10 2x5

11 prime

12 2x2x3

13 prime

14 2x7

It is actually quite easy to find a gap of **at least** any specified length using the command “factorial”, denoted “!”. Let’s look at an example of doing this.

Suppose I want a gap of at least five composite numbers between two primes.

I take 5 + 1 = 6

I use the numbers that I calculate this way

6! + 2 = 722

6! + 3

6! + 4

6! + 5

6! + 6 = 726

Here are 5 adjacent composite numbers. Let’s look at how I know they are composite.

6! + 2 = 6x5x4x3x2x1 + 2…factor out the 2’s = 2 (6x5x4x3x1 + 1).

This is not factored to primes probably, but it is factored! It is a composite then.

6! + 3 factors to 3 (6x5x4x2x1 + 1).

The next 3 factor the same way!

719 is the largest prime below the list, and 727 is the smallest prime larger than the list. NOTE: I got these two primes off a list of primes not from my memory!

So the gap is actually 7 composites…which is AT LEAST 5.

Let’s abstract an algorithm for finding a gap between two primes of size *n*.

* Get the gap size; call it *n*.
* Using (*n* + 1)!, add the numbers 2, 3, 4, …, *n* + 1 to the factorial in succession to create the composites between the primes.
* Use a list of primes to find the one below and the one above.

Why the list? Because the Gap Algorithm only guarantees

* AT LEAST *n* composites. There might be more than *n* composites between. It does not guarantee that the list created is the
* first gap of that size either!

Like so much about primes, it is a small thing and we really don’t know everything about gaps yet.

So a rehearsal for the homework. Suppose we want a gap of size 81.

Take 82. Use 82! + 2, 82! + 3, 82! + 4, … , 82! + 82 as the composites. Use a list of primes to find the nearest one smaller than 82! + 2 and the smallest one larger than 82! + 82

**Popper 02 Question 5**

PRIME FREE Sequences

An additional effort to master the “pattern” of the primes was work with sequences. An arithmetic sequence is a numbered list of elements that follow a given addition pattern. The initial number is given, call it *a*, (element number 1) and each subsequent sequence element is found by adding a given number, *d*, to the preceding one.

For example *a* = 5 and *d* = 2

5, 7, 9, 11, 13, 15,…, ,…

Note that this sequence has both primes and composites in it.

 is prime while is composite

Now it is possible to create a “prime-free” arithmetic sequence, believe it or not.

If *a* and *d* are NOT relatively prime, then the sequence will be prime free.

 (5, 10) = 5…not relatively prime

5, 15, 15, 25, 35…each is divisible by 5 and thus composite.

This was a little factlet discovered by a person working to define the pattern of the primes…back in the day before there was a proof that there is not a pattern…

TWIN PRIMES! In addition to gaps of any length considerable study has gone into gaps of size 1. When two prime numbers are one apart on the number line, they are called twin primes. 3 and 5 are twin primes, 17 and 19 are twin primes.

Another pair is (41, 43). It is not known if there are an infinite number of twin primes. There is a LOT of research going on about these numbers.

**Popper 02 Question 6**

**Lucky numbers**

Now, still looking at primes, let’s look at an ancient prime-finding device:

The Sieve of Eratosthenes

Eratosthenes was a Greek mathematician who lived quite a bit of time before Christ was born. He invented a foolproof way for students to find primes if they knew their multiplication tables.

Make a list of all natural numbers

01 02 03 04 05 06 07 08 09

10 11 12 13 14 15 16 17 18

Ignore 1 – it’s just a place holder.

Circle 2 and then cross out all multiples of 2 that follow.

Circle 3 and cross out all multiples of 3 that follow. Skip up to 5, circle it and cross out all multiples of 5 that follow. Skip to 7, and so forth. The only numbers that end up NOT crossed off are the primes.

In 1955 at Los Alamos, during a coffee break, the research group led by Stanislaw Ulam visited a bit about fun math ideas. They started with the Sieve and changed the cross out rules. They decided that they’d cross out numbers based on POSITION rather than factors.

They made a list of the natural numbers. Circled 1 and crossed out every other number. Then they went to 3 and circled it and crossed out every third number of the remaining numbers. They then circled 7 and canceled every 7th number of the remaining numbers. They called the remaining numbers the Lucky Numbers.

They got a set {1, 3, 7, 9, 13, 15, 21, 25, 31, …} that is shockingly like the Primes!

All of the Luckies are odd (all except one of the primes are odd). Luckies are a bit different from primes in that some Luckies are prime (3, 7, 13…) and some Luckies are composite (15, 21, 25,…) and 1 is a Lucky and neither prime nor composite while some Luckies are prime . Luckies and UnLuckies do form a partition on the natural numbers, though. A new and different partition than you’ve seen before.

The spaces between the Luckies get further and further apart as you get bigger.

For the primes there’s a famous conjecture called Goldbach’s Conjecture:

Every even number larger than 2 is expressible as the sum of 2 primes (not necessarily distinct e.g. 4 = 2 + 2)

There’s a matching conjecture for the Luckies: Every even number is the sum of 2 Luckies (not necessarily distinct e.g. 2 = 1 + 1).

Neither has been proved yet…that’s why they are “conjectures” and not theorems!

In fact, even on small details the sets are similar. There are the famous “twin primes” – prime numbers separated by exactly one composite number (3, 5; 17, 19;…)

And there are “twin Luckies” (7, 9; 13, 15;…)

Both sets of “twins” might be infinite in size…or might not.

The Luckies have been studied ever since!

Up until the Luckies, primes were considered to be unique – no other set was quite like them…but now, the Luckies demonstrate that there may indeed be similar sets! We just need to find them.

There’s a really cool interactive display on Wikipedia that will let you watch the crossing out pattern on the Luckies. Be sure to check it out if you have time!

<http://en.wikipedia.org/wiki/Lucky_number>

**Popper 02 Question 7**

**Video 2**

Repunits

Base 2 and Base 2 Repunits

Mersenne Primes

Repunits and Mersenne Primes connected

Popper 02, continued Questions 8 - 10

Now, just naturally mathematicians got into playing with natural numbers. Early in the 1800’s someone started playing around with number strings of one.

You know them,

1, 11, 111, 1111, 11111, …

There are an infinite number of these. How do you know that?

By 1966, there were enough people playing around with these, that Dr. Albert Beiler gave them a name:

Repunit repeated unit

SOME repunits are prime, while others are composite, and there’s that 1, too:

Set Diagram:

How can you tell which is which? Well, there’s a way to tell for SURE which are composite.

Not so long ago, someone came up with subscripts rather than writing all those ones. And someone else noticed a pattern with the indices and the state of the number. See if you can figure it out





What’s the pattern? Let’s take some time and see if we can see it!

There’s even a formula for repunits.



The study of repunits bloomed with the advances in computers! Factoring primes is still hard work…someone will find a “probably prime” repunit and it takes a couple of YEARS to factor it! For example,  was called “probably prime” in about 1966; it took until 1977 to PROVE it prime!

Now all of this was about base 10 primes.

But there are other bases for numbers, remember?

…how many of you are familiar with binary? Also known as Base 2

We can review binary: digits {0, 1}

 the first base 2 repunit is also one in base 10



 a base 2 repunit that is a prime in base 10



1012 =

Popper 02 Question 8

“There are 10 kinds of people in the world…those who know binary and those who don’t.”

Computer science tee shirt.

Let’s make a number line with base 10 on top and base 2 on the bottom

And let’s list both names for some base 2 repunits:

**Popper 02 Question 9**

Base 2 repunit primes are called Mersenne Primes. Let’s unpack that by talking about Mersenne Primes first – these are natural base 10 numbers. Then we’ll tie in base 2 repunits.

Mersenne numbers and Mersenne prime numbers

Many early (16th century) mathematicians felt that all the numbers of the form 2*n*-1 were prime for *all* primes *n*,

(NOT all *n*, just prime *n*’s)



Let’s look at what they saw:

but in 1536 Hudalricus Regius showed that 211-1 = 2047 was not prime (it is 23times 89).  So it is a Mersenne number but not a Mersenne prime number.

By 1603 [Pietro Cataldi](http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Cataldi.html) had correctly verified that 217-1 and 219-1 were both prime, but then incorrectly stated 2*n*-1 was also prime for 23, 29, 31 and 37, used as exponents in the formula.

In 1640 [Fermat](http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Fermat.html) showed Cataldi was wrong about 23 and 37 as exponents. Then in 1738, Euler showed Cataldi was also wrong about 29.  [Sometime later](http://primes.utm.edu/notes/by_year.html#31) Euler showed Cataldi's assertion about 31 was correct.

Enter French monk [Marin Mersenne](http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Mersenne.html) (1588-1648).  Mersenne stated in the preface to his *Cogitata Physica-Mathematica* (1644) that the numbers 2*n*-1 were prime for

*n* = 2, 3, 5, 7, 13, 17, 19, **31, 67, 127 and 257**

and were composite for all other positive integers *n* < 257 From 2 to 257 is called “Mersenne’s Range”.  Mersenne's (incorrect) conjecture fared only slightly better over the centuries than Regius', but still got his name attached to these numbers.

We know of 37 Mersenne primes so far.

**Definition:** When 2*n*-1 is prime it is said to be a **Mersenne prime**. When it is NOT prime, it is called a **Mersenne number**.

Remember that the exponent needs to be a prime number NOT a composite.

It was obvious to Mersenne's peers that he could not have tested all of these numbers (in fact he admitted as much), but they could not test them either.  It was not until over 100 years later, in 1750, that Euler verified the next number on Mersenne's and Regius' lists, 231-1, was prime.

After another century, in 1876, [Lucas](http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Lucas.html) verified 2127-1 was also prime.  Seven years later Pervouchine showed 261-1 was prime, so Mersenne had missed this one.  In the early 1900's Powers showed that Mersenne had also missed the primes 289-1 and 2107-1.

\*source: Wikipedia

Finally, by 1947 Mersenne's range, *n* < 258, had been completely checked and it was determined that the correct list is:

*n* = 2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107 and 127. Now these are the exponents, not the Mersenne Primes themselves…

Let’s look at a list of the Mersenne Primes:

The table below lists some Mersenne primes from the internet:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **#** | ***p*** | ***Mp*** | ***Mp* digits** | **Discovered** | **Discoverer** | **Method used** |
| 1 | 2 | [3](http://en.wikipedia.org/wiki/3_%28number%29) | 1 | c. 430 BC | [Ancient Greek mathematicians](http://en.wikipedia.org/wiki/Greek_mathematics)[[15]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-Philolaus-15) |  |
| 2 | 3 | [7](http://en.wikipedia.org/wiki/7_%28number%29) | 1 | c. 430 BC | Ancient Greek mathematicians[[15]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-Philolaus-15) |  |
| 3 | 5 | [31](http://en.wikipedia.org/wiki/31_%28number%29) | 2 | c. 300 BC | Ancient Greek mathematicians[[16]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-Euclid-16) |  |
| 4 | 7 | [127](http://en.wikipedia.org/wiki/127_%28number%29) | 3 | c. 300 BC | Ancient Greek mathematicians[[16]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-Euclid-16) |  |
| 5 | 13 | 8191 | 4 | 1456 | Anonymous[[17]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-primepages-17)[[18]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-18) | [Trial division](http://en.wikipedia.org/wiki/Trial_division) |
| 6 | 17 | 131071 | 6 | 1588[[19]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-19) | [Pietro Cataldi](http://en.wikipedia.org/wiki/Pietro_Cataldi) | Trial division[[20]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-20) |
| 7 | 19 | 524287 | 6 | 1588 | Pietro Cataldi | Trial division[[21]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-21) |
| 8 | 31 | [2147483647](http://en.wikipedia.org/wiki/2147483647) | 10 | 1772 | [Leonhard Euler](http://en.wikipedia.org/wiki/Leonhard_Euler)[[22]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-22)[[23]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-23) | Enhanced trial division[[24]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-24) |
| 9 | 61 | 2305843009213693951 | 19 | 1883 November[[25]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-25) | [I. M. Pervushin](http://en.wikipedia.org/wiki/Ivan_Mikheevich_Pervushin) | [Lucas sequences](http://en.wikipedia.org/wiki/Lucas_sequence) |
| \*\*10 | 89 | 618970019…449562111 | 27 | 1911 June[[26]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-26) | [R. E. Powers](http://en.wikipedia.org/wiki/R._E._Powers) |  |
| 11 | 107 | 162259276…010288127 | 33 | 1914 June 1[[27]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-27)[[28]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-28)[[29]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-29) | [R. E. Powers](http://en.wikipedia.org/wiki/R._E._Powers)[[30]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-30) | Lucas sequences |
| 12 | 127 | 170141183…884105727 | 39 | 1876 January 10[[31]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-31) | [Édouard Lucas](http://en.wikipedia.org/wiki/%C3%89douard_Lucas) | Lucas sequences |
| 13 | 521 | 686479766…115057151 | 157 | 1952 January 30[[32]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-autogenerated4-32) | [Raphael M. Robinson](http://en.wikipedia.org/wiki/Raphael_M._Robinson) | [LLT](http://en.wikipedia.org/wiki/Lucas%E2%80%93Lehmer_primality_test) / [SWAC](http://en.wikipedia.org/wiki/SWAC_%28computer%29) |
| 14 | 607 | 531137992…031728127 | 183 | 1952 January 30[[32]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-autogenerated4-32) | Raphael M. Robinson | LLT / SWAC |
| 15 | 1,279 | 104079321…168729087 | 386 | 1952 June 25[[33]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-33) | Raphael M. Robinson | LLT / SWAC |
| 16 | 2,203 | 147597991…697771007 | 664 | 1952 October 7[[34]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-autogenerated2-34) | Raphael M. Robinson | LLT / SWAC |
| 17 | 2,281 | 446087557…132836351 | 687 | 1952 October 9[[34]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-autogenerated2-34) | Raphael M. Robinson | LLT / SWAC |
| 18 | 3,217 | 259117086…909315071 | 969 | 1957 September 8[[35]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-35) | [Hans Riesel](http://en.wikipedia.org/wiki/Hans_Riesel) | LLT / [BESK](http://en.wikipedia.org/wiki/BESK) |
| 19 | 4,253 | 190797007…350484991 | 1,281 | 1961 November 3[[36]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-autogenerated1-36)[[37]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-autogenerated5-37) | Alexander Hurwitz | LLT / [IBM 7090](http://en.wikipedia.org/wiki/IBM_7090) |
| 20 | 4,423 | 285542542…608580607 | 1,332 | 1961 November 3[[36]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-autogenerated1-36)[[37]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-autogenerated5-37) | Alexander Hurwitz | LLT / IBM 7090 |
| 21 | 9,689 | 478220278…225754111 | 2,917 | 1963 May 11[[38]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-autogenerated3-38) | [Donald B. Gillies](http://en.wikipedia.org/wiki/Donald_B._Gillies) | LLT / [ILLIAC II](http://en.wikipedia.org/wiki/ILLIAC_II) |
| 22 | 9,941 | 346088282…789463551 | 2,993 | 1963 May 16[[38]](http://en.wikipedia.org/wiki/Mersenne_prime#cite_note-autogenerated3-38) | Donald B. Gillies | LLT / ILLIAC II |

This is from Wikipedia, too

Let’s look at this chart closely…do you see where the chartmakers ran out of space because of the size of the Mersenne Primes?

Now let’s look at some base 2 numbers and expand them and see this connection!



Let’s take a moment to unpack the change from base 2 to base 10, too.

Note that Mersenne Primes are a proper subset of the Primes. Let’s draw the set diagram.

How will we work in the base 2 repunits? Who would have guessed base 2 repunits would even be in a set of primes!

Let’s review the connections.

Somebody started looking a repunits and then named them. Then folks noticed an interesting connection between the subscripts and the type of number it was, prime or composite.

Then somebody with an interest in numbers other than base 10 noticed repunits in base 2 and found yet another, more complicated connection right back to primes again in base 2, but only a certain proper subset of the primes, the Mersenne primes.

And notice how things have gone throughout time…MUCH older work is being woven into newer work with living mathematicians.

Wrapping it up now

Now let’s talk about the number 3.

What are all the things we know about 3?

Now let’s look at a special website. One of my favorites is Number Gossip.

http://www.numbergossip.com/list



Let’s explore some of these properties, using the definitions from Number gossip:

Common Properties of 3

Deficient

The number n is *deficient* if the sum of all its positive divisors except itself is less than n.

Evil

The number n is *evil* if it has an even number of 1's in its binary expansion.

Lucky

To build the *lucky* number sequence, start with natural numbers. Delete every second number, leaving 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, ... . The second number remaining is 3, so delete every third number, leaving 1, 3, 7, 9, 13, 15, 19, 21, ... . The next number remaining is 7, so delete every 7th number, leaving 1, 3, 7, 9, 13, 15, 21, ... . The next number remaining is 9, so delete every ninth number, etc.

Those numbers were lucky they weren't crossed out.

* [1](http://www.numbergossip.com/1),
* **3**,
* [7](http://www.numbergossip.com/7),
* [9](http://www.numbergossip.com/9),
* [13](http://www.numbergossip.com/13),
* ...

The lucky number "sieve" was developed by the legendary Stanislaw Ulam, who was born and died on the 13th of the month (lucky number) at the age of 75 (lucky number)... and his birth year of 1909 saw the release of a film called “The Lucky Number”. Coincidence?

Odd

A number is *odd* if it is not divisible by 2.

Numbers that are not odd are [even](http://www.numbergossip.com/list#even_numbers). Compare with another pair -- [evil](http://www.numbergossip.com/list#evil_numbers) and [odious](http://www.numbergossip.com/list#odious_numbers) numbers.

* [1](http://www.numbergossip.com/1),
* **3**,
* [5](http://www.numbergossip.com/5),
* [7](http://www.numbergossip.com/7),
* [9](http://www.numbergossip.com/9),
* ...

Palindrome

A *palindrome* is a number that reads the same forward or backward.

* [1](http://www.numbergossip.com/1),
* [2](http://www.numbergossip.com/2),
* **3**,
* [4](http://www.numbergossip.com/4),
* [5](http://www.numbergossip.com/5),
* [6](http://www.numbergossip.com/6),
* ...

Prime

A *prime* is a positive integer greater than 1 that is divisible by no positive integers other than 1 and itself.

Prime numbers are opposite to [composite](http://www.numbergossip.com/list#composite_numbers) numbers.

Square-free

A number is said to be *square-free* if its prime decomposition contains no repeated factors.

* [1](http://www.numbergossip.com/1),
* [2](http://www.numbergossip.com/2),
* **3**,
* [5](http://www.numbergossip.com/5),
* [6](http://www.numbergossip.com/6),
* [7](http://www.numbergossip.com/7),
* ...

Triangular

If you start with n points on a line, then draw n-1 points above and between, then n-2 above and between them, and so on, you will get a triangle of points. The number of points in this triangle is a *triangle* number.

* [1](http://www.numbergossip.com/1),
* **3**,
* [6](http://www.numbergossip.com/6),
* [10](http://www.numbergossip.com/10),
* [15](http://www.numbergossip.com/15),
* ...

Twin Prime

A prime number is called a *twin* prime if there exists another prime number differing from it by 2.

* **3**,
* [5](http://www.numbergossip.com/5),
* [7](http://www.numbergossip.com/7),
* [11](http://www.numbergossip.com/11) …

Ulam

The next *Ulam* number is uniquely the sum of two earlier distinct Ulam numbers.

* [1](http://www.numbergossip.com/1),
* [2](http://www.numbergossip.com/2),
* **3**,
* [4](http://www.numbergossip.com/4),
* [6](http://www.numbergossip.com/6),
* [8](http://www.numbergossip.com/8),
* ...

Fibonnaci

*Fibonacci* numbers are numbers that form the Fibonacci sequence. The Fibonacci sequence is defined as starting with 1, 1 and then each next term is the sum of the two preceding ones.

Fibonacci numbers are [very common in nature](http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html). For example, a pineapple has 8 spirals if you count one way, and 13 if you count the other way.

* [1](http://www.numbergossip.com/1),
* [1](http://www.numbergossip.com/1),
* [2](http://www.numbergossip.com/2),
* **3**,
* [5](http://www.numbergossip.com/5),
* [8](http://www.numbergossip.com/8),
* [13](http://www.numbergossip.com/13),
* ...

Mersenne

A number of the form 2p - 1 is called a *Mersenne* number if p is [prime](http://www.numbergossip.com/list#primes).

It was believed many years ago, that all Mersenne numbers are prime. This is not so, thus there is a separate entry for [Mersenne prime](http://www.numbergossip.com/list#mersenne_primes) numbers.

* **3**,
* [7](http://www.numbergossip.com/7),
* [31](http://www.numbergossip.com/31),
* [127](http://www.numbergossip.com/127),
* ...

Mersenne Prime

[Mersenne prime](http://www.numbergossip.com/3)

A [Mersenne](http://www.numbergossip.com/list#mersenne_numbers) number which is also prime is called a *Mersenne prime*.

The drive to find prime numbers among Mersenne numbers supplies humanity with the largest known prime numbers.

* **3**,
* [7](http://www.numbergossip.com/7),
* [31](http://www.numbergossip.com/31),
* [127](http://www.numbergossip.com/127),
* ...

**Popper 02 Question 10**

Homework

1. How old are you? Write that number down and then go to Number Gossip (a fabulous internet site for budding number theory enthusiasts) and write down all the words that describe the number that is your age. Some will be familiar and some won’t. Take just one of the new descriptors, write down the definition and write a short paragraph or two on how your age meets that definition.

If you’re shy about writing out your real age, use the age you feel most of the time!

2. (Q, +) is a group. (Q without zero, multiply) is a group, too. Write a brief essay on how multiplying two integers is an illustration of closure in this second group. Then select two carefully chosen rational numbers and discuss how they illustrate that there are inverses in this second group.

Why is teaching addition and multiplication just illustrating that these two groups are closed over and over again?

3. Why is (14, 15) = 1? Translate the words and write a brief paragraph about why this is true.

4. Find a gap of at least 4 composites between two primes. List all 4 composites and the 2 primes numbers in order and show your work in agonizing detail.

5. Create your own prime free sequence and discuss how you know there are no primes in it even though you can’t write the whole list down to check.

6. Find a Lucky Number between 50 and 100. Discuss how you found it and give two other properties of this number.

7. Convert  to a base 10 number. Show your work in detail. Give 3 other properties of this number.

8. Can you tell if  is prime or composite? How do you what you know about it?